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Young Researchers' Meeting

Geometrically nonlinear Cosserat elasticity - How nonlinearities in the models PDEs influence regularity

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The overall goal of this talk is to give an overview of the progress achieved regarding this topic within the last six years.

Since the beginning of the 1900s, linear Cosserat elasticity is well known in the engineering community for modelling micro-polar elastic solids. But from a mathematician's perspective a geometrically nonlinear version of the model is interesting. Existence of solutions for the latter has been known for about 20 years, while regularity questions were investigated only very recently, after recognizing structures closely related to (p-) harmonic maps, a topic well known in the mathematics community.

Starting with the Cosserat bulk model, we present some different regularity results, for Cosserat energy minimizers as well as for critical points. It turns out that classical regularity theory for (p-) harmonic maps into manifolds is an essential tool in deriving those results. At the same time, the geometric nature of the model's nonlinearities in some cases allows not only regular, but also quite singular solutions to exist. We demonstrate some geometric ideas behind those.

Finally, singular solutions do not exist, when we go away from the (3d-) Cosserat bulk model towards a (2d-) Cosserat model for shells, both for initally flat or curved midsurfaces.

An analysis of thermo-electro-mechanical material behaviour across scales

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The macroscopic response of electrically conductive materials is inherently governed by their microstructural characteristics and by the interactions among thermal, electrical, and mechanical fields. Interfaces such as grain or phase boundaries play a decisive role in this context, acting as preferential sites for degradation and contributing to the overall resistivity of the material.

Analytical homogenisation techniques, based on asymptotic expansions and two-scale convergence, are employed to derive the governing macroscopic equations for thermo-electrically coupled behaviour. The analysis demonstrates that the effective relations, previously introduced on physical grounds through Hill–Mandel-type arguments, arise naturally within the rigorous two-scale convergence framework, thereby providing a firm theoretical basis for the effective description of conductive materials.

The Hill–Mandel-type computational homogenisation scheme is shown to yield the same effective relations as the analytical framework, confirming consistency between the two approaches. Building upon this verified computational foundation, the formulation is extended to microstructures containing cohesive-type interfaces, enabling the investigation of interface-induced anisotropy and size effects within a first-order setting. Energetic consistency between the micro- and macroscales is maintained, and the approach is validated through analytical benchmarks and representative numerical examples [1].

Overall, the developments presented provide a unified and thermodynamically consistent framework for the multiscale analysis of coupled thermo-electro-mechanical processes in heterogeneous materials, with direct implications for non-destructive diagnostics and the design of advanced functional conductors.

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Modelling the viscoelastic behaviour of Alginate-Gelatine Hydrogels

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Hydrogels are widely being used in cell-culture to provide a scaffolding medium, allowing cells to grow in a 3D environment. Typically, 3D-printing is used not only to layer the different cell-types but also to produce perfused physical structures that can lead to miniaturised models of human organs [1], e.g. lungs, livers, intestines.

The application has a mechanical nature since: a) printing through a small nozzle induces viscous forces and can kill the cells prematurely; b) viscosity allows better control of the structure geometry and the structures themselves can be subjected to internal pressure loading; and c) the viscoelastic properties of the hydrogel are known to determine cell survival rate in the culture and the establishment of a functioning extracellular matrix [2].

Cyclically-loaded samples of alginate-gelatine hydrogels under indentation show that the material responds viscoelastically, including relaxation and internal friction, similar to many polymers. The material is very compliant and undergoes large deformations at even very moderate loads.

Presented is a homogeneous, objective, finite-strain material model inspired by the microstructure, as observed under electron microscope. The contact problem of the indentation ball is solved by the recently-published Latent Variable Proximal Point algorithm [3]. The state space, extended by the elastic left Cauchy-Green Tensor, is discretised via a mixed finite element approach and allows the model to be parameterised to reproduce the experimentally-observed material responses.

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A variational model for fused silica up to 20 GPa

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Compaction mechanisms of fused silica under hydrostatic pressure are often described using the equation of state (EOS). As a function of pressure depending on the densification ratio, this has a sigmoidal shape and can be interpreted as a phase transition between two phases. While in between, microstructure can arise through mixing processes.
We present a variational model which leads to the well-known pictures from experimental physics over a simple binary mixture term. Based on a previous work [1], the resulting phases are expressed as generalized convex Young measures to maintain a continuous microstructure evolution. In addition, a dissipation potential described there is applied, which consists of a stochastic matrix associated with the so-called dissipation distance.
First results obtained using 2D FEM correspond well with other studies and are presented in this context.

References

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Scaling laws for a singularly perturbed, incompatible, geometrically linear two-well energy

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Motivated by the analysis of microstructures in shape-memory alloys, we study scaling laws for a singularly perturbed, geometrically linear two-well energy. In this problem, it is typically assumed that the stress-free strains are compatible (that is, they differ by symmetrized rank-one matrix) and that the prescribed boundary data are also compatible (that is, they are a convex combination of the stress-free strains).

The focus of this talk is to understand the behavior of optimal microstructures when these compatibility assumptions are violated. To this end, we analyze the higher order behavior of the singular perturbation model, building on the work of Kohn [2], who studied the incompatible setting without surface energy.

This analysis yields the following result: In two dimensions, if the weight ϵ of the surface energy is small and the boundary data enforces oscillations then the minimal energy scales either as $\epsilon^{\frac{2}{3}}$ or as $\epsilon^{\frac{4}{5}}$. This agrees with the scaling laws known from the compatible setting in the geometrically linear theory [3] and the nonlinear theory [1]. The upper bounds are established via branching constructions, while the lower bounds follow from a Fourier-based argument. The presented results, which will appear in an upcoming publication, are based on my master's thesis under the supervision of Angkana Rüland and Antonio Tribuzio.

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On Symmetric Quasiconvexification and Its Application toward Phase Transformations

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In the theory of linear elasticity and elastoplasticity, one considers energies of the form

$$\int_\Omega F(e(u))\,\mathrm{d}x,$$

which depend on the symmetric gradient $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ rather than on ∇u. Such energies require a reformulation of the coercivity and lower semicontinuity properties necessary for applying the direct method. For lower semicontinuity, it is necessary to work with the symmetric analogue of quasiconvexity of the energy densities. To understand the microstructure formation in composite materials, where non-quasiconvex energy densities arise naturally, one needs to determine the relaxation of these densities. Similarly to the classical case, one strategy for determining these envelopes is to construct an upper bound (symmetric rank-one convex envelope) and a lower bound (symmetric polyconvex envelope) and to show that they coincide. These symmetric notions of semi-convexity differ greatly from their classical counterparts. For instance, the determinant function is polyconvex but not symmetric polyconvex. In this talk, we present an overview of the symmetric notions of semi-convexity and we characterize the symmetric polyconvex envelope in three different ways, using a Carathéodory formula, Legendre conjugation, and the translation method. These characterizations of the symmetric polyconvex envelope are based on the characterization of symmetric polyconvex functions established in [1]. We apply these characterizations to determine the symmetric polyconvex envelope of the function

$$F(\varepsilon) = \min\{(\operatorname{tr}\varepsilon)^2 + \tfrac{1}{2}|\operatorname{dev}\varepsilon|^2 + 1, 2(\operatorname{tr}\varepsilon)^2 + |\operatorname{dev}\varepsilon|^2\}$$

in $\mathbb{R}^{2\times 2}_{\mathrm{sym}}$, which describes a phase transformation in an elastic medium. Finally, we show that the resulting function already coincides with the symmetric quasiconvex envelope.

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Quantitative homogenization of geometric motions through a random field of obstacles in arbitrary dimensions

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Consider the evolution of sets by forced mean curvature flow through a field of random obstacles. Such an evolution is commonly used to model a combination of a uniform forcing, surface tension, and random material impurities. The effective large scale behaviour is expected to be a first order motion. However, previous results heavily relied on the assumption that there is a global minimum speed of expansion and hence on the absence of any actual obstacles.

We obtain a quantitative homogenization result even with actual obstacles, potentially allowing the interface to get stuck locally, eventually leading to enclosures behind a main front. So far in this regime not even a qualitative stochastic homogenization result had been available. The existence of a global minimum speed is replaced with a probabilistic assumption. We assume that on large scales with high probability 'fat' sets can be approximated from within by sets which are increasing with respect to the evolution and expand at an 'effective' minimum speed - that is, we ignore smaller enclosures left behind the main front. This assumption is satisfied for example if the obstacles are distributed according to a Poisson point process with low enough intensity.

Solving Interface Problems On Microscopic Composite Materials

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Fuel cells are characterized by a complex microscopic structure of a composite material. Modelling the ion exchange at the interface is numerical challenging. The complex microscopic structure leads into a highly oscillating solution.

To overcome the computational challenges of directly resolving the microstructure, we apply a homogenization technique based on the asymptotic expansion approach. By separating macroscopic and microscopic scales, we derive a cell problem and a macroscopic problem.

An important aspect of this work is the treatment of interface discontinuities. Both, the solutions of the microscopic problem and the cell problem exhibit discontinuities at material interfaces. To accurately model these discontinuities, we use the extended finite element method (XFEM). The implentation of XFEM in a threee dimensional framework is challenging. Finally, the results of the microscopic and macroscopic models are compared, highlighting the effectiveness of the homogenized approach and the XFEM framework in capturing the multiscale behavior of the system.

Influence of Orientation and Latent Hardening on Deformation Patterning in Single Crystal Plasticity

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Deformation patterning due to the alternating activation of different sets of slip systems is in many cases the energetically favorable response of a single crystal to an applied deformation. The onset and evolution of these deformation bands greatly influences texture formation, and consequently, the feasibility of metal forming processes.

An example where deformation pattern has been observed is the rolling of nickel aluminide (Ni_3Al), a typically brittle intermetallic phase. This material has beneficial properties such high strength at elevated temperature, comparably low density, high creep resistance, and high corrosion resistance. These properties make it ideal for special applications where such scenarios are found, namely blades in gas turbines and jet engines.

Motivated by experimental studies that reveal a strong influence of crystallographic orientation on deformation patterning and plasticity in single crystal Ni_3Al, we study systematically the influence of interaction parameters for latent hardening and crystallographic orientation on pattern formation in fcc single crystals. In particular, we look at their effect on the deformation behavior in plane strain compression rolling. This study is carried out numerically using *DAMASK*[1], a multiphysics crystal plasticity simulation package, where a viscoplastic phenomenological model is used to describe plasticity.

The results confirm the influence of crystallographic orientation on pattern formation and plastic deformation. It is also found that different assumptions for latent hardening, as described by the interaction matrix of the employed crystal plasticity model, can promote or demote deformation patterning.

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Main Meeting

Modeling and analysis of multiscale problems in geodynamics

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Geodynamics of Solid Earth is concerned with the motion and deformation of rock-like materials on time scale ranging from minutes (for Earth quakes) to millions of years (for tectonic drifts). As the motion behaves fluidic on long time scales but elastoplastic on shorter ones, it is natural to formulate the equations in Eulerian/spatial setting. We present the typical features of some relevant thermodynamically consistent models based on GENERIC and Lie derivatives for the relevant quantities like momentum, plastic distortion, internal stresses, and scalar internal variables (water content, "the aging parameter", ...) [2].

Existence results are obtained for suitably simplified or regularized models only, see [1, 5]. A spatially one-dimensional model for the shear motion between two rigid (tectonic) plates is analyzed qualitatively in [3]. Using a variant of the Dieterich-Ruina (1980-1987) rate-and-state friction law it can be shown that shear profiles localize. Looking at symmetrized profiles the regularization can be avoided, leading to an explicitly solvable model for steady shear profiles. However, numerical evidence suggests that these shear flows may be unstable leading to a periodic stick-slip motion which is called "episodic tremor and slip" in geoscience. Such motions are well documented, e.g., between the American and the Pacific plates (near Victoria Bay, period of 16 months, and 5 mm displacement per period).

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Homogenization and optimization of resonances in photonic crystals

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It is planned to review briefly the state of art in Applied Physics studies of high-Q optical cavities and to compare it with the mathematical nonselfadjoint spectral optimization for electromagnetic and acoustic wave equations. Connections with random resonances, homogenization, and (non-)unique continuation for 3-dimensional dissipative Maxwell systems will be discussed following the recent joint paper with Matthias Eller [1]. If time permits, we consider also the computation of TEM-resonances of minimal decay in 1-D photonic crystals with the use of the optimal control approach that was developed jointly with Herbert Koch and Ievgen Verbytskyi [2].

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Symmetries in stochastic homogenization

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We study the role of symmetry in stochastic homogenization, specifically focusing on the apparent material tensor and its corresponding fluctuations. Within the framework of the RVE method, we examine how the underlying symmetry of an ensemble dictates the invariance of the material tensor [1, 2], and how the choice of unit cell may compromise this invariance [3]. To address this issue, we propose a symmetry-informed post-processing technique based on orthogonal projection and demonstrate its ramifications through large scale computational FFT-based homogenization.

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Interface conditions for Maxwell's equations by homogenization of thin inclusions: transmission, reflection or polarization

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We consider the time-harmonic Maxwell equations in a complex geometry. Of particular interest are geometries that model polarization filters or Faraday cages. We study the situation that the underlying domain contains perfectly conducting inclusions, the inclusions are distributed in a periodic fashion along a surface. The periodicity is $\eta > 0$ and the typical scale of the inclusion is η, but we allow also the presence of even smaller scales, e.g. when thin wires are analyzed. We are interested in effective interface conditions resulting in the limit $\eta \to 0$. Depending on geometric properties of the inclusions, the effective system can imply perfect transmission, perfect reflection or polarization [1].

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Deep learning-aided inverse design of porous metamaterials

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The ultimate aim of the study is to explore the inverse design of porous metamaterials using a deep learning-based generative framework. Specifically, we develop a property-variational autoencoder (pVAE), a variational autoencoder (VAE) augmented with a regressor, to generate structured metamaterials with tailored hydraulic properties, such as porosity and permeability. While this work uses the lattice Boltzmann method (LBM) to generate intrinsic permeability tensor data for limited porous microstructures, a convolutional neural network (CNN) is trained using a bottom-up approach to predict effective hydraulic properties. This significantly reduces the computational cost compared to direct LBM simulations. The pVAE framework is trained on two datasets: a synthetic dataset of artificial porous microstructures and CT-scan images of volume elements from real open-cell foams. The encoder-decoder architecture of the VAE captures key microstructural features, mapping them into a compact and interpretable latent space for efficient structure-property exploration. The study provides a detailed analysis and interpretation of the latent space, demonstrating its role in structure-property mapping, interpolation, and inverse design. This approach facilitates the generation of new metamaterials with desired properties.

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Generative machine learning for crystallographic texture representation

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Polycrystalline materials consist an ensemble of grains, and the distribution of their lattice orientations is described as the crystallographic texture. Texture is one of the main factors governing mechanical and functional properties. The evolution of texture is highly sensitive to processing parameters, such as deformation rate, thermal history, strain path etc., making its accurate prediction central to microstructure-sensitive process design. Conventionally, texture is represented through the orientation distribution function (ODF), a probability density over the rotation group SO(3) which is a non-Euclidean manifold. We introduce a generative modeling framework based on normalizing flows that leverages bidirectional transformations to map simple latent distributions to complex target distributions, making them well suited for capturing the crystallographic texture. This approach enables both unconditional sampling of complex textures and conditional modeling, enabling both process-structure linkage and inverse design through optimization. The method is able to capture the multimodal distributions and offers mapping between processing conditions and resulting texture. This framework underscores the potential of generative machine learning to advance materials design and microstructural analysis in non-Euclidean domains.

A variational damage-plasticity model at finite strains

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The consideration of ductile fracture at finite strains involves different challenges, such as accounting for large deformations, geometric non-linearities and the coupling of damage and plasticity. This work aims to provide a variational damage-plasticity model at finite strains with damage represented by a phase-field variable. The coupled damage-plasticity problem is formulated in the framework of rate-independent problems, where it is represented by a stored energy functional and a dissipation potential [1].

Solutions of the variational formulation are obtained by considering a natural time discretisation, which leads to a sequence of minimisation problems to be solved. Such a minimisation problem consists of smooth as well as non-smooth non-convex functionals. The formulation of the damage-plasticity problem as a rate-independent system in the sense of [1] nevertheless allows the construction of a numerical solver based on a proximal Newton method coupled with robust and efficient multigrid methods. This approach, which can be applied to different plasticity models, ensures convergence, replaces the classical predictor-corrector scheme for the plastic strain, and offers a reduced computation time [2].

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Convex continuum damage models with softening via optimal transport

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We present a class of isotropic damage models in continuum mechanics which involve convex potentials and permit to reproduce strain softening behavior. We derive these models by computing the convex envelope of a rate-limited simple damage model, finishing the agenda proposed by Schwarz et al. [1].

References

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The generalized elasticity of mechanical metamaterials

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When used in bulk applications, mechanical metamaterials set forth a multiscale problem with many orders of magnitude in scale separation between the micro and macro scales. Direct numerical simulations of mechanical metamaterials are prohibitively expensive due to the strict separation between the lattice and the structural scales. Hence, multiscale continuum analysis, specifically *discrete-to-continuum methods*, suggests itself as a means of characterizing the effective properties of metastructures at the structural scale. However, mechanical metamaterials fall outside conventional homogenization theory on account of the flexural, or *bending*, response of their members, including torsion. Notwithstanding, we show that homogenization theory based on calculus of variations and notions of Gamma-convergence can be extended to account for bending. The great advantage of these methods is that they determine, *ansatz-free*, the functional form of the limiting continuum energy. In this manner, the homogenized metamaterials are shown to exhibit intrinsic *generalized elasticity* in the continuum limit. By exploiting the quadratic-form structure of the discrete energies, Gamma-convergence additionally supplies closed-form expressions for all effective properties. Examples of two and three-dimensional metamaterials, including honeycomb and octet-truss lattices, are presented. The convergence of the discrete energy to the continuum limit is also illustrated by means of numerical examples. To zeroth order, the continuum limit of metamaterials is *micropolar*, with both displacement and rotational degrees of freedom, but exhibits no size effect. To higher order, the overall energetics of the metastructure can be characterized explicitly in terms of the solution of the zeroth-order continuum problem by the method of *Gamma-expansion*. When applied to fracture mechanics, the analysis predicts that the discreteness of the lattice results in *anti-shielding* of crack tips, i.e., *coarser is weaker*, in agreement with recent experimental observations.

Recent Advances in Data-Driven Inelasticity

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Data-driven mechanics continues to show great promise for simulations of highly complex material behavior at substantially reduced computational cost. Extending the framework to inelasticity, however, poses far-reaching and intricate challenges. Our general approach employs a history surrogate that stores the essential information from a material's loading history, together with a propagator that updates this information. In this way, the original distance function of Kirchdörfer & Ortiz can be augmented by the history surrogate, while the core algorithm remains almost unchanged. A significant step toward generalization is achieved by using a neural network as both history surrogate and propagator. Important intermediate milestones in our project include replacing an RNN (recurrent neural network) with an LSTM, and ultimately with an LSCM; these transitions will be a particular focus of the talk. Finally, we propose a seemingly small but essential modification to the original algorithm that has a pronounced impact on the accuracy and stability of the data-driven scheme – an effect that can be illustrated clearly using simulations of elastic material behavior. Taken together, these advances provide encouraging evidence that, under certain conditions, data-driven mechanics can achieve strong performance on 2D inelastic problems.

Multiscale modeling of magnetic shape memory alloy behavior: Energy relaxation, phase-field and surrogate models

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Magnetic shape memory alloys (MSMAs) are multifunctional, usually single-crystalline, materials whose unique properties are directly linked to microstructure evolution and multi-physical coupling effects. Early models mainly relied on concepts adopted from phenomenological plasticity theory [**?**]. More recently, approaches of energy relaxation have proven to very successfully reproduce the main inelastic, nonlinear, anisotropic, magnetomechanical features of MSMAs [**?**]. They build on computing energetic hulls to non-convex free energy landscapes whose wells define preferred states of straining and magnetization. Mathematically, such models have roots in the so-called constrained theory of magnetoelasticity developed by DeSimone and James, which itself combines the Ball and James theory of microstructure formation with classical micromagnetics. These energy relaxation models have further successfully been implemented in mixed finite element formulations that not only account for MSMA constitutive behavior, but solve the coupled static equilibrium and magnetostatic Maxwell's equations. Our current focus lies on leveraging these theoretical and numerical tools to build a consistent multiscale framework that allows informing computationally efficient system-level models (e.g., of Preisach-type) for real-time operations. To this end, a recently proposed scale-bridging methodology, see [**?**], is discussed in detail.

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Phase-field approximation of grain-boundary energies accounting for lattice symmetry

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I will present a phase field approximation for sharp interface energies, defined on partitions, as appropriate for modeling grain boundaries in polycrystals. The label takes value in $\mathcal{O}(d)/G$, where G is the point group of a lattice, the phase-field approximation fully respects the symmetry. These functionals can be used for the simulation of grain growth or for image reconstruction of grain boundaries. The talk is based on joint work with Vito Crismale, Adriana Garroni and Annalisa Malusa (Sapienza, Roma).

Computing the elastic fluctuation tensor of random microstructures

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The apparent properties of random microstructures computed on cells of finite size fluctuate randomly. Duerinckx et al. [1] showed that for specific ensembles describing thermal conductivity problems, these statistical fluctuations are quantified by the fluctuation tensor. We apply the same approach to linear elasticity, introducing the *elastic fluctuation tensor* to quantify the stochastic fluctuation of the apparent stiffness of microstructure volume elements.

Using Fast-Fourier-Transform (FFT) based computational homogenization techniques, we compute the elastic fluctuation tensor of example microstructural ensembles. As this tensor is of order eight interpreting its components is not straightforward. We derive efficient fluctuation tensor representations by taking into account statistical symmetries of the ensemble [2].

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Efficient modelling of plates with internal structure using a Cosserat model

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We investigate efficient modelling of highly structured plates. The components and internal structures are converted into a phenomenological description via the plate's mean surface, i.e. a double manifold. As in the Reissner–Mindlin formulation, we introduce internal rotational degrees of freedom to describe kinematics. However, unlike Reissner–Mindlin theory, we omit the symmetry operator for strain and curvature measures. Consequently, four independent bending moments occur in the present plate theory.

This need already arises from the potentially asymmetrical reinforcement, but the internal structure reinforces it. We propose a method to determine the stiffness of the internal structure using few parameters and integrate it into plate theory. It becomes evident that plate theory with four independent bending stress components accommodates a broader range of internal configurations.

This leads to an asymmetrical stress tensor in the mechanical model, raising the question of consistency with moment balance. It becomes clear that this can no longer represent a local continuum and that non-locality introduces an internal length parameter [1].

Such panels may help the construction industry reduce concrete use by substituting materials like wood. To protect wood from moisture and weathering, it is embedded between mineral cover layers. The focus lies on small wood pieces between mineral inner structures. Both panel thickness and wood-piece dimensions relate to the internal length of this plate formulation.

Combining concrete and wood enables a CO2-neutral or even CO2-storing building product. Yet high-quality wood is scarce, further motivating these efforts. Much small wood can only be processed into secondary products (chipboards, etc.) unsuitable for load-bearing or weather-exposed elements. Hence, combining wood with concrete best exploits the strengths of both materials.

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Modeling wave propagation and boundary effects in acoustic metamaterials by a relaxed micromorphic continuum

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Exploring the dynamical response of mechanical metamaterials within the framework of elastic wave propagation has gathered increasing attention in recent decades. To accurately describe elastic wave propagation in these materials – characterized by their heterogeneities or microstructures – it is essential to adopt non-local or enriched modeling approaches.

In this context, we propose the relaxed micromorphic model (RMM), which effectively captures these phenomena by treating the material as continuum with enriched kinematics. A crucial micro-mechanical effect addressed by this model is the presence of band gaps, for which wave with specific frequencies cannot propagate through the material. Our previous studies have demonstrated that the RMM adequately describes the bulk behavior of mechanical metamaterials.

Challenges arise, however, when considering finite-size metamaterial blocks, where boundary effects can significantly influence behavior due to reduced dimensions. Consequently, it becomes necessary to enhance the relaxed micromorphic model framework to account for these boundary effects [1, 2, 3]. We will illustrate how incorporating boundary forces facilitates accurate modeling of finite-size metamaterial specimens within the framework of micromorphic elasticity.

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The true stress true strain monotonicity in isotropic hyperelasticity

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In this talk I will present recent findings concerning stability questions in isotropic hyperelasticity augmenting more traditional approaches as Legendre-Hadamard elasticity. The concept is connected to a notion of stress increases with strain. The logarithmic strain tensor appears prominently. For incompressible hyperelasticity the condition reduces to Hills inequality.

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The energy scaling behaviour for singular perturbation problems of staircase type in linearized elasticity

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In the talk, we discuss scaling laws for singular perturbation problems of "staircase type" within the geometrically linearized theory of elasticity. More precisely, we focus on a three-well problem and show that the scaling depends both on the lamination order of the prescribed Dirichlet boundary data and on the number of (non-)degenerate symmetrized rank-one directions in the symmetrized lamination convex hull. Our analysis is based on localization techniques in Fourier space. A preprint of the corresponding article is expected to be uploaded within the next two months.

Continuum Models for Short Fibre Reinforced Concrete

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Short fibre composites are becoming increasingly popular in many applications. This is also true for civil engineering, where short fibre cementitious composites are used more often. For the use in load bearing structures, two challenges need to be addressed: 1) a constitutive mapping is necessary to calculate the design load and to predict cracking behaviour and 2) the fiber orientations need to be predicted and controlled during the casting of the fresh concrete. To address 1) a constitutive mapping based on the use of isotropic tensor functions of the strain tensor and the orientation tensor is proposed and to address 2) numerical simulations of fiber concrete casting are presented.

Thermo-mechanically coupled FE-FFT-based simulation framework for polycrystalline shape memory alloys

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The behavior of polycrystalline shape memory alloys is characterized by solid-solid phase transformations between austenite and martensite. These complex microstructural effects lead to promising material properties, such as shape memory, pseudo-elasticity, and pseudo-plasticity. However, they also present challenges regarding computational predictability. Unlike classical steel, phase transformations in these alloys are driven not only by thermal loading, but also by mechanical stress. The model presented in this talk incorporates a variational approach based on [1] into a thermo-mechanically coupled FE-FFT-based simulation framework [2]. This two-scale simulation approach combines a finite element calculation to analyze macroscopic material behavior with a Fast Fourier transformation-based method at the microscopic scale. It serves as a computationally efficient alternative to the classical FE^2 approach. In addition to presenting numerical results on the behavior of SMAs under thermo-mechanical loading, we will demonstrate further developments in modeling phase transformations in our research group using a variational approach for temperature-gradient dependent phase transformations.

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